Fibre Bragg Gratings

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This document describes the analysis of fibre bragg gratings by coupled mode theory and transfer matrix theory. These methods are used in the OptiSystem FBG Sensor component.

Contents

1 Definitions

The optical fibre or waveguide is presumed to have one mode of a fixed polarization. This document follows the usual coordinate system of optical waveguides. The propagation is in the direction of the Z axis. The X and Y axes form the basis of the transverse plane. The waveguide consists of a structure of variable refractive index in the transverse plane, but with no variation in Z. On such a structure, the solution of the time-harmonic Maxwell equations separate with the Z coordinate. For example, typical electromagnetic field components in the transverse plane of a wave travelling from left to right (positive Z) could be of the form

$$\mathbf{E}_t(x, y, z) = \mathbf{e}_t(x, y)e^{-j\beta z} \tag{1}$$

In equation (1), $\mathbf{e}_t(x, y)$ is the field distribution of the fundamental mode in the transverse plane, and β is the propagation constant of the fundamental mode. The propagation constant, β, is related to the modal index, n_{modal}, as

$$\beta = k\ n_{modal} \tag{2}$$

where k is the free space wavenumber

$$k = \frac{2\pi}{\lambda} \tag{3}$$

A grating is a periodic variation of the refractive index (or permittivity) in the direction of propagation. These variations are usually applied at the fibre core for maximum effect. Suppose the refractive index of the core to be n_0, and suppose this refractive index is varied above and below this value by the quantity Δn. Let $f(z)$ be a periodic function with period Λ . Suppose $f(z)$ varies anywhere between -1 and +1. The variation of the permittivity in the grating can be written as

$$\Delta\epsilon(x, y, z) = 2P(x, y)\ f(z)A(z)\ n_0\ \Delta n \tag{4}$$

n_0 is the refractive index before the application of the grating. The function $P(x, y)$ indicates the position of the grating in the transverse plane. It takes the value 1.0 in places where the grating has been applied, and zero outside of this. For example, $P(x, y)$ could be 1.0 for x and y inside the core of the fibre, and zero elsewhere. Δn is the maximum deviation (greater or smaller) of the refractive index due to the grating. $A(z)$ is the apodization, a slowly varying function that is used to modulate the index modulation, Δn. $f(z)$ indicates the shape of the grating in the propagation direction. It takes a value between -1 and + 1 to modulate the maximum deviation, Δn. For example, for a grating made of multilayers, this would be a rectangular-shaped function. For fibre Bragg gratings made from UV exposure, the shape is often a circular function, such as sine or cosine. In any case, in this work $f(z)$ is composed as a cosine series,

$$f(z) = 2\sum_{n=1}^{\infty} F_n \cos nKz \tag{5}$$

$$K = \frac{2\pi}{\Lambda} \tag{6}$$

As such, $f(z)$ can be any continuous function that is symmetric about the centre of the grating period interval. It is possible to find the coefficients in the expression from the Fourier integral:

$$F_n = \frac{1}{\Lambda}\int_0^{\Lambda} f(z)\cos Knz\; dz \tag{7}$$

$f(z)$ is a real-valued function, so the F_n coefficients are real.

2 Electromagnetics

Maxwell's equations can be specialized to the case of waveguides. The waveguide is characterized by the permittivity, ϵ, being independent of propagation, i.e. ϵ is not a function of Z. The permittivity of the waveguide can be represented as a function of transverse coordinates only,

$$\epsilon(x, y) \tag{8}$$

On the other hand, after the grating is applied, there will be variation of permittivity in the direction of propagation, but of a specific form that involves $f(z)$. Call $\tilde{\epsilon}$ the permittivity after the grating has been applied to the waveguide. This permittivity is the sum of the waveguide permittivity and the deviations defined by $\Delta\epsilon$:

$$\tilde{\epsilon}(x, y, z) = \epsilon(x, y) + \Delta\epsilon(x, y, z) \tag{9}$$

Where $\Delta\epsilon$ is given by (4). When the magnetic field is eliminated from the time independent form of Maxwell's equations, the electric field remains and follows

$$\nabla^2\mathbf{E} + k^2\tilde{\epsilon}(x, y, z)\mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) \tag{10}$$

Propagation on waveguides can be simplified by separating the longitudinal (Z) coordinate from the transverse coordinates. The field and operator are written as vector sums

$$\mathbf{E} = \mathbf{E}_t + E_z\hat{\mathbf{z}} \qquad \nabla = \nabla_t + \hat{\mathbf{z}}\frac{\partial}{\partial z} \tag{11}$$

Substitution in (10) leads to

$$\nabla^2\mathbf{E}_t + k^2\tilde{\epsilon}(x, y, z)\mathbf{E}_t = \nabla_t(\nabla_t \cdot \mathbf{E}_t + \frac{\partial E_z}{\partial z}) \tag{12}$$

The last term has a londitudinal component of the electric field, but it can be almost eliminated by considering the divergence equation:

$$\nabla \cdot (\epsilon\mathbf{E}) = 0 \tag{13}$$

(13) separates into longitudinal and transverse parts as

$$\nabla_t^2 \cdot (\tilde{\epsilon}\mathbf{E}_t) + \frac{\partial\tilde{\epsilon}}{\partial z}E_z + \tilde{\epsilon}\frac{\partial E_z}{\partial z} = 0 \tag{14}$$

However, $\partial\epsilon/\partial z \ll \epsilon$ because the variation in z is only from the grating, and $\Delta\epsilon \ll \epsilon$. Therefore the longitudinal term in (12) can be replaced with the first term of (14) to get

$$\frac{\partial \mathbf{E}_t}{\partial z^2} + k^2\tilde{\epsilon}(x,y,z)\mathbf{E}_t = \nabla_t(\nabla_t \cdot \mathbf{E}_t - \frac{1}{\tilde{\epsilon}}\nabla_t \cdot (\tilde{\epsilon}\mathbf{E}_t)) - \nabla_t^2\mathbf{E}_t \tag{15}$$

an equation with only the transverse field components as dependent variables. When $\Delta n = 0$ (when there is no grating), the operators after the first term are all independent of z. In this case the electromagnetic fields will be a combination of modes. The longitudinal variable, Z, separates and a left to right travelling wave solution can be written as

$$\mathbf{E}_t(x,y,z) = \mathbf{e}_t(x,y)e^{-j\beta z} \tag{16}$$

where $\mathbf{e}_t(x,y)$ is the mode field distribution, and β is the propagation constant of that mode. The mode field is normalized as

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathbf{e}_t(x,y) \cdot \mathbf{e}_t(x,y)\; dxdy = 1 \tag{17}$$

Modes are mathematically equivalent to eigenvectors, and the propagation constant is related to an eigenvalue. To see them from this point of view, it is enough to rearrange (15) as

$$\frac{\partial^2 \mathbf{E}_t}{\partial z^2} = -k^2\tilde{\epsilon}(x,y,z)\mathbf{E}_t + L(\mathbf{E}_t) \tag{18}$$

The terms collected into L form a linear differential operator. That operator has the permittivity $\tilde{\epsilon}$ in both the numerator and denominator. Therefore the operator is not changed very much by replacing $\tilde{\epsilon}$ with ϵ. Putting the modal wave (16) into (18) for the waveguide with no grating (permittivity ϵ) gives

$$\beta^2\mathbf{e}_t(x,y)e^{-j\beta z} = k^2\epsilon(x,y,z)\mathbf{e}_t(x,y)e^{-j\beta z} - L(\mathbf{e}_t(x,y)e^{-j\beta z}) \tag{19}$$

Written this way, the right travelling wave in (16) is interpreted as an eigenvector of the operator

$$k^2\epsilon(x,y,z) - L \tag{20}$$

and the eigenvalue is β^2. A similar wave travelling to the left, $\mathbf{e}_t(x,y)e^{j\beta z}$, is also an eigenvector of β^2, since

$$\beta^2\mathbf{e}_t(x,y)e^{j\beta z} = k^2\epsilon(x,y,z)\mathbf{e}_t(x,y)e^{j\beta z} - L(\mathbf{e}_t(x,y)e^{j\beta z}) \tag{21}$$

2.1 Slowly Varying Envelope Approximation

When a grating is applied, the optical field in the transverse plane does not change, but the very small periodic reflections from the grating interfaces can sometimes add up to a significant total reflection. Therefore a solution is sought with these properties. The transverse variation is still thought to be proportional to $\mathbf{e}_t(x,y)$, as in (16), but there will need to be two waves, one going from left to right and the other going the other way that was created from the reflections:

$$\mathbf{E}_t(x,y,z) = a(z)\mathbf{e}_t(x,y)e^{-j\beta z} + b(z)\mathbf{e}_t(x,y)e^{j\beta z} \tag{22}$$

The amplitude of the right travelling wave is $a(z)$, and the amplitude of the left travelling (reflected) wave is $b(z)$. The wave amplitudes are functions of the propagation distance, z, because the grating causes continuous transfer of optical power from one wave to the other. The influence of the grating over a few wavelengths is very small, it is significant only when accumulating over many wavelengths. Therefore (22) is a slowly varying envelope approximation (SVEA) as well. $a(z)$ and $b(z)$ are functions of z, but they do not change significantly over one wavelength. The rapid variations are taken up by the exponential functions.

To find the equations governing the two unknown wave amplitudes $a(z)$ and $b(z)$, put (22) in the left side of (18):

$$\frac{\partial^2 \mathbf{E}_t}{\partial z^2} = \begin{array}{lll} a''(z)e^{-j\beta z}\mathbf{e}_t(x,y) & -\,2j\beta a'(z)e^{-j\beta z}\mathbf{e}_t(x,y) & -\beta^2 a(z)e^{-j\beta z}\mathbf{e}_t(x,y) \\ +\,b''(z)e^{j\beta z}\mathbf{e}_t(x,y) & +\,2j\beta b'(z)e^{j\beta z}\mathbf{e}_t(x,y) & -\beta^2 b(z)e^{j\beta z}\mathbf{e}_t(x,y) \end{array} \tag{23}$$

When (22) is put in the right hand side of (18), and expanding the permittivity as in (9),

$$\frac{\partial^2 \mathbf{E}_t}{\partial z^2} = \begin{array}{cc} -k^2\epsilon(x,y,z)a(z)\mathbf{e}_t(x,y)e^{-j\beta z} & -k^2\epsilon(x,y,z)b(z)\mathbf{e}_t(x,y)e^{j\beta z} \\ -k^2\Delta\epsilon(x,y,z)a(z)\mathbf{e}_t(x,y)e^{-j\beta z} & -k^2\Delta\epsilon(x,y,z)b(z)\mathbf{e}_t(x,y)e^{j\beta z} \\ +a(z)L(\mathbf{e}_t(x,y)e^{-j\beta z}) & +b(z)L(\mathbf{e}_t(x,y)e^{j\beta z}) \end{array} \tag{24}$$

When the right hand sides of (23) and (24) are set equal, there are a number of terms that cancel, owing to the eigenvector relations (19) and (21). After eliminating these terms and writing the right and left going waves separately:

$$0 = \begin{array}{l} \mathbf{e}_t(x,y)e^{-j\beta z}\left[a''(z) - 2j\beta a'(z) + k^2\Delta\epsilon(x,y,z)a(z)\right] \\ +\,\mathbf{e}_t(x,y)e^{j\beta z}\left[b''(z) + 2j\beta b'(z) + k^2\Delta\epsilon(x,y,z)b(z)\right] \end{array} \tag{25}$$

Next the vector property and dependence on x and y is removed by taking the inner product with the vector mode field $\mathbf{e}_t(x,y)$ and integrating over the entire transverse plane. Most factors reduce to 1 because of the normalization of the vector mode (17). The exception is third term, since $\Delta\epsilon(x,y,z)$, defined in (4), depends on transverse coordinates that are independent of the mode field distribution. To accommodate this independence, define the overlap integral

$$\Gamma = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(x,y)\,\mathbf{e}_t(x,y)\cdot\mathbf{e}_t(x,y)\;dxdy \tag{26}$$

With this definition, (25) reduces to an ordinary differential equation:

$$\begin{array}{c} e^{-j\beta z}[a''(z) - 2j\beta a'(z) + 2k^2 n_0\Delta n\Gamma f(z)\;a(z)] \\ +\,e^{j\beta z}[b''(z) + 2j\beta b'(z) + 2k^2 n_0\Delta n\Gamma f(z)\;b(z)] = 0 \end{array} \tag{27}$$

3 Coupled Mode Equations

The equations (27) can be solved approximately by neglecting terms that will be much smaller than the rest. For example, since $a(z)$ abd $b(z)$ are slowly varying, their second derivatives are much smaller than the other terms and can be neglected. Multiplying (27) by $e^{j\beta z}$ will give an equation that can estimate the rate of change of $a(z)$

$$-2j\beta a'(z) + 2k^2 n_0 \Delta n \Gamma f(z) e^{2j\beta z}\, b(z) = 0 \tag{28}$$

The second and third terms were ignored because they are rapidly oscillating with z, and therefore will not contribute when the equation is integrated with z to calculate $a(z)$. The last term is retained because, although both $e^{2j\beta z}$ and $f(z)$ are rapidly varying, there is a possibility that the product $e^{2j\beta z} f(z)$ might be slowly varying in some conditions. To see this, use the definition of $f(z)$ in (5) to expand the terms

$$\begin{aligned} e^{2j\beta z} f(z) &= e^{2j\beta z}\left[F_1 e^{jKz} + F_1 e^{-jKz} + F_2 e^{2jKz} + F_2 e^{-2jKz} ...\right] \\ &= F_1 e^{j(2\beta+K)z} + F_1 e^{j(2\beta-K)z} + F_2 e^{j(2\beta+2K)z} + F_2 e^{j(2\beta-2K)z} ... \end{aligned} \tag{29}$$

For example, $e^{2j\beta z} f(z)$ could be a significant influence in (28) if 2β took a value close to K, the wavenumber of the grating. The optical wavelength that leads to the condition $2\beta = K$ is called the Bragg wavelength, the first order response of the grating. Looking through the expansion, it is apparent that $e^{2j\beta z} f(z)$ could be slowly varying, and therefore influential, when the optical wavelength makes 2β close to $2K$ instead of K. The latter condition is the second order response of the grating. If the function $f(z)$ has a Fourier coefficient F_2 that is not zero, then the grating will show reflection at that wavelength too. And so on for all the higher order terms.

It is convenient to define a detuning, δ, as a measure of how close 2β is to one of these wavenumbers. For example, to investigate the first order grating spectrum, define the detuning as

$$\delta = 2\beta - K = 2\pi\left(\frac{2n}{\lambda} - \frac{1}{\Lambda}\right) \tag{30}$$

where n is the waveguide modal index. The maximum response occurs at the Bragg wavelength, λ_B, when δ is zero:

$$\lambda_B = 2n\Lambda \tag{31}$$

To investigate the first order grating spectrum, let δ vary over a range of size K centered at zero. With this small value of δ, only the second term on the right hand side of (29) is significant, and (28) reduces to

$$a'(z) = -jk\Delta n \Gamma F_1 e^{j\delta z} b(z) \tag{32}$$

(in (32) the ratio n_0/n is considered to be close to 1.)

A similar approach can be used to estimate the rate of change of $b(z)$. The second derivatives in (27) are ignored, and then the equation is multiplied by $e^{-j\beta z}$. When the rapidly varying terms are neglected, the remaining terms are

$$2j\beta b'(z) + 2k^2 n_0 \Delta n \Gamma e^{-2j\beta z} f(z) a(z) = 0 \tag{33}$$

The slowly varying term of $e^{-2j\beta z} f(z)$ is $e^{-j\delta z} F_1$, and the equation for the evolution of $b(z)$ becomes

$$b'(z) = jk\Delta n \Gamma F_1 e^{-j\delta z} a(z) \tag{34}$$

Noting that the overlap integral, modulation index, and first order Fourier coefficient all contribute in the same way, it is more convenient to gather them together in one number, the coupling coefficient, γ:

$$\gamma = k\Gamma F_1 \Delta n \tag{35}$$

This definition leads to the most compact form of the coupled mode equations. These clearly show the interaction of forward and backward waves:

$$\begin{aligned} \frac{da}{dz} &= -j\gamma e^{j\delta z}\, b(z) \\ \frac{db}{dz} &= j\gamma e^{-j\delta z}\, a(z) \end{aligned} \tag{36}$$

3.1 Solution of the Coupled Mode Equations

The Coupled Mode Equations (CME) (36) can be easily solved for the case of uniform gratings. If the grating properties, such as modal index, grating period, index modulation, and so on, do not change with z, the solution can be constructed from elementary functions. The basic properties of the solution can be seen by eliminating $b(z)$ from (36):

$$a''(z) - j\delta a'(z) - \gamma^2 a(z) = 0 \tag{37}$$

Letting $a(z) = e^{sz}$ will show some properties of the possible solutions. The s must satisfy

$$s^2 - j\delta s - \gamma^2 = 0 \tag{38}$$

which has two possible solutions

$$s_1 = j\delta/2 + \sqrt{\gamma^2 - \delta^2/4}$$

and

$$s_2 = j\delta/2 - \sqrt{\gamma^2 - \delta^2/4} \tag{39}$$

If the detuning, δ, is less than 2γ, the solution for the right travelling wave, $a(z)$, will vary with z exponentially. This means that this wave must vanish after at least some length of propagation, and the grating is reflecting optical power at this wavelength. The range of wavelengths at which the grating reflects optical power is called the stopband. The condition at the edges of the band are

$$\delta^2 = 4\gamma^2 \tag{40}$$

which happens for the detunings

$$\delta_1 = 2\gamma \quad \text{and} \quad \delta_2 = -2\gamma \tag{41}$$

The optical wavelengths at these two detunings are

$$\lambda_1 = \frac{2\pi n\Lambda}{\pi + \gamma\Lambda} \tag{42}$$

$$\lambda_2 = \frac{2\pi n\Lambda}{\pi - \gamma\Lambda} \tag{43}$$

Optical wavelengths in the range $\lambda_1 < \lambda < \lambda_2$ (the stopband) may be partly transmitting and partly reflecting for a grating of some length. However, if the grating is made longer it will tend to become not transmitting and reflecting all optical power.

3.2 Stopband

In the stopband, the real part of s is written

$$q = \sqrt{\gamma^2 - \delta^2/4} \tag{44}$$

The coupled mode equations are an initial value problem. If the right and left wave amplitudes are given at one end of the grating, e.g. $z = 0$, the CME may be used to find the waves at the other end of the grating ($z = L$). Suppose the two wave amplitudes at $z = 0$ are known and given by

$$a(0) = a_0 \tag{45}$$

$$b(0) = b_0 \tag{46}$$

The first CME gives a relation between b_0 and the rate of change of a at that point

$$a'(0) = -j\gamma b_0 \tag{47}$$

When (37) is solved, the initial values (45) and (47) will specify a unique value for $a(z)$:

$$a(z) = e^{j\delta z/2}\left[a_0 \cosh qz - \left(\frac{\gamma b_0}{q} + \frac{\delta a_0}{2q}\right) j \sinh qz\right] \tag{48}$$

To find $b(z)$, find the derivative of (48)

$$a'(z) = \frac{j\delta}{2} e^{j\delta z/2} \left[a_0 \cosh qz - \left(\frac{\gamma b_0}{q} + \frac{\delta a_0}{2q} \right) j \sinh qz \right] \tag{49}$$

$$+ e^{j\delta z/2} \left[a_0 q \sinh qz - \left(\frac{\gamma b_0}{q} + \frac{\delta a_0}{2q} \right) jq \cosh qz \right] \tag{50}$$

and substitute in the first CME

$$b(z) = \frac{j}{\gamma} e^{-j\delta z} \frac{da}{dz} \tag{51}$$

to get

$$b(z) = \frac{j}{\gamma} \frac{j\delta}{2} e^{-j\delta z/2} \left[a_0 \cosh qz - \left(\frac{\gamma b_0}{q} + \frac{\delta a_0}{2q} \right) j \sinh qz \right] + \frac{j}{\gamma} e^{-j\delta z/2} \left[a_0 q \sinh qz - \left(\frac{\gamma b_0}{q} + \frac{\delta a_0}{2q} \right) jq \cosh qz \right] \tag{52}$$

Apparently these solutions for the waves $a(z)$ and $b(z)$ are all exponential functions of qz. Therefore the grating in the stopband will transmit some light if the grating is sufficiently short. However, as $z \to \infty$, the transmitted optical power will have to go to zero. For this reason the range $\lambda_1 < \lambda < \lambda_2$ is called the stopband.

3.3 Passband

Optical wavelengths outside the range $\lambda_1 < \lambda < \lambda_2$ are in the passband. In this case, both terms in (39) are imaginary. For the passband, define

$$p = \sqrt{\delta^2/4 - \gamma^2} \tag{53}$$

Solving for $a(z)$ and $b(z)$ as in the stopband gives similar expressions, with the hyperbolic functions of qz replaced by circular functions of pz

$$a(z) = e^{j\delta z/2} P_1 \tag{54}$$

$$b(z) = -\frac{\delta}{2\gamma} e^{-j\delta z/2} P_1 + \frac{j}{\gamma} e^{-j\delta z/2} P_2 \tag{55}$$

where

$$P_1 = \left[a_0 \cos pz - \left(\frac{\gamma b_0}{p} + \frac{\delta a_0}{2p} \right) j \sin pz \right] \tag{56}$$

$$P_2 = \left[-a_0 p \sin pz - \left(\frac{\gamma b_0}{p} + \frac{\delta a_0}{2p} \right) jp \cos pz \right] \tag{57}$$

Since $a(z)$ and $b(z)$ are oscillating instead of exponential, in the passband the grating is transmitting most of the optical power.

4 Non Uniform Gratings

If one or all of the grating properties (grating period, index modulation, modal index ...) are changing with z, the solutions of the CME given so far will not apply. To manage the non-uniform case, the grating is divided into a sequence of short uniform gratings, each one having properties approximately equal to the local values of the non-uniform grating.

The transfer matrix will give the wave amplitudes at one end of a grating given the amplitudes at the other end. The solutions (48), (52), (54), and (55) will find $a(L)$ and $b(L)$ from their values at $z = 0$, a_0 and b_0. The transfer matrix is defined

$$\begin{bmatrix} a(L) \\ b(L) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} \tag{58}$$

The first column of the transfer matrix is found by setting $a_0 = 1$ and $b_0 = 0$. The second column is found by setting $a_0 = 0$ and $b_0 = 1$. The nonuniform grating is divided into n_{seg} segments of equal length, Δz

$$\Delta z = \frac{L}{n_{seg}} \tag{59}$$

each of these segments is taken to be a uniform grating, with constant grating parameters. A transmission matrix, $\mathbf{T}_i$ is found for each uniform segment, whose grating parameters are equal to the average value of the grating parameters at the corresponding location in the nonuniform grating. The transmission matrix of the entire grating is found by matrix multiplication of all the transmission matrices of the individual segments

$$\mathbf{T} = (\mathbf{T}_{n_{seg}})(\mathbf{T}_{n_{seg}-1})...\mathbf{T}_3\mathbf{T}_2\mathbf{T}_1 \tag{60}$$

The spectrum of the whole grating can be found from the transfer matrix that is the product of all the grating segment transfer matrices.

4.1 Chirp

For example, the grating period might not be the same over the whole grating. Sometimes optical gratings are ”chirped”, the local period is one value at one end ($z = 0$) , but the period changes continuously over the length of the grating so that the local period has a different value at the other end ($z = L$). The local period, $\Lambda(z)$ for the chirped grating is

$$\Lambda(z) = \Lambda_0 - \frac{z - L/2}{L}\lambda_c \tag{61}$$

where Λ_0 is the period of the unchirped grating and λ_c is the level of chirp.

4.2 Apodization

The level of index modulation is sometimes graded over the length of the grating. The apodization can make the index modulation less when its value is less than 1, as indicated in (4). It can be applied with a Gaussian function, as

$$A(z) = \exp\left\{-\ln 2\left[\frac{2(z-L/2)}{sL}\right]^2\right\} \tag{62}$$

where s is the Gaussian apodization parameter.

5 Sensors

The spectra of Fibre Bragg gratings can be sensitive to environmental conditions, such as temperature or mechanical strain. This section describes how temperature and strain modify the physical parameters, and therefore how the spectrum will be modified by strain or temperature.

5.1 Temperature Sensor

Increase or decrease in temperature from a reference level will cause a change in the refractive index of the material of the fibre. Since the temperature change is expected to be applied equally in the transverse plane, the change is expected to be applied to the modal index, n

$$\Delta n = \xi n \Delta T \tag{63}$$

where ξ is the thermo-optic coefficient of the glass in the fibre and ΔT is the temperature change from the reference temperature.

In addition to the thermooptic effect, the grating period is also affected by the elongation of the fibre from the thermal expansion coefficient

$$\Delta\Lambda = \eta\Lambda(T - T_{ref}) \tag{64}$$

5.2 Strain Sensor

Strain is a mechanical stretching of the fibre along its axis. Therefore the grating period is directly affected as

$$\Delta\Lambda = \epsilon\Lambda \tag{65}$$

As with temperature, strain also affects refractive index. The relation is quantified by the elastooptic parameters, P_{11} , P_{12} and the Poisson ratio, ν. The refractive index is affected as

$$\Delta n = -\frac{1}{2}n^3\epsilon[P_{12} - \nu(P_{11} + P_{12})] \tag{66}$$

When strain and temperature change are applied to the grating, the grating parameters are modified accordingly. The level of strain or temperature change can be deduced from observing a change in the grating spectrum. The most noticeable effect on the spectrum is a shift in the maximum reflection / minimum transmission. Temperature and strain can cause a shift in the Bragg wavelength.