# Multimode Optical Fiber

## 1 Objective

Determine the optical modes that exist for multimode step index fibers and investigate their performance on optical systems.

## 2 Pre-Lab

The backbone of optical systems is optical fiber. It allows for the propagation of very short pulses, which translates to high bit rates, extremely long distances while experiencing modest signal degradation. Understanding how signals propagate in a fiber allows for better control and optimization of an optical system.

From the principle of total internal reflection it is understood that rays of light can be guided by propagating them through a region of high index of refraction surrounded by a region of low index of refraction. If the rays have a larger angle of incidence than the critical angle the light will be reflected back into the core region.

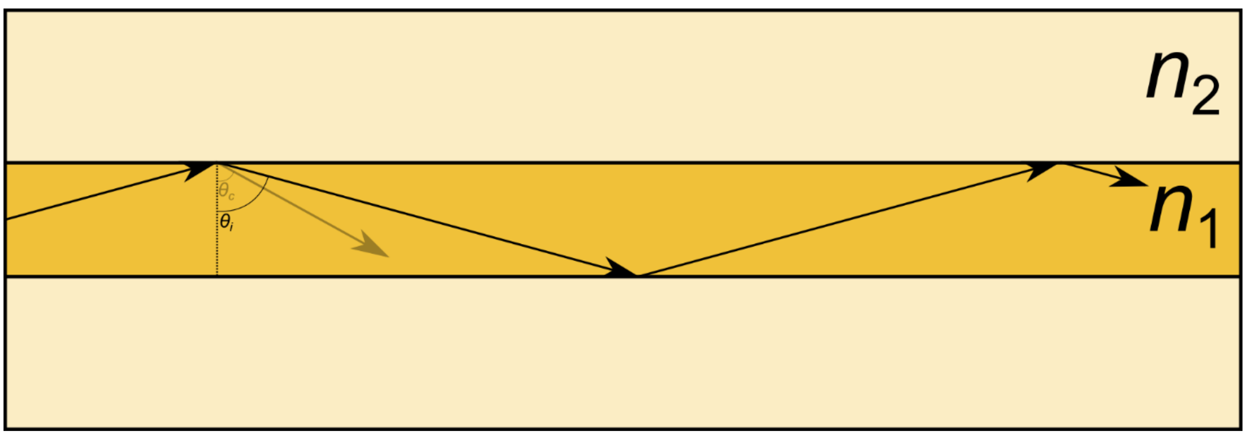



*Figure 1: Rays with a larger angle of incidence are kept confined to the high index core of a multimode waveguide.*

In this example, the core region $n_1$ has a higher index than the outer cladding region $n_2$. There then exists a critical angle, $\theta_c$, such that rays incident at a greater angle will be completely reflected. This is a useful approximation for large core fibers, but when the core radius becomes comparable to the wavelength of the light the wave nature dominates. The Maxwell equations must then be used to solve for the guiding conditions.

Starting from the Maxwell equations in cylindrical coordinates and continuing on from the Helmholtz equation the problem can be stated as a Bessel differential equation of the form:

$$\frac{d^2E_z(r)}{dr^2} + \frac{1}{r}\frac{dE_z(r)}{dr} + \left(\gamma^2 + k_0^2\varepsilon_r - \frac{m^2}{r^2}\right)E_z(r) = 0, \quad (1)$$

In cylindrical coordinates, the *z* component of the electric field as a function of the radius *r*, can be related to the propagation constant γ, the electric permittivity $\varepsilon_r$, and an integer *m* which represents the periodic conditions for the ϕ component of the electric field.

Assuming lossless media, γ = jβ, the solution of this differential equation are Bessel functions. In addition, approximating as a weakly guided fiber $n_1 \approx n_2$, and matching the fields at the boundary between the core and cladding the solution can be expressed by a characteristic equation.

$$X\frac{J_{m\pm1}(X)}{J_m(X)} = \pm\frac{K_{m\pm1}(Y)}{K_m(Y)}. \quad (2)$$

The solutions to the characteristic equation provide the propagation constant and the field profiles for the $LP_{mn}$ modes. For a more exhaustive derivation please refer to the reference [1]. In equation 2, *J* and *K* are the Bessel functions of the first kind and the modified Bessel functions of the second kind. The values *X* and *Y* are given by:

$$X = a\sqrt{k_0^2 n_1^2 - \beta^2}, \quad (3)$$

$$Y = a\sqrt{\beta^2 - k_0^2 n_2^2}, \quad (4)$$

where *a* is the radius of the core. We can also a variable called the V parameter defined by:

$$X^2 + Y^2 = V^2, \quad (5)$$

$$V = 2\pi\frac{a}{\lambda}\sqrt{n_1^2 - n_2^2}. \quad (6)$$

For optical fiber with large V parameters the number of modes can be approximated by:

$$M = \frac{4}{\pi^2}V^2. \quad (7)$$

<u>Questions:</u>

2.1 From Snell's Law derive the expression for the critical angle.

Answer: $\sin(\theta_c) = \frac{n_2}{n_1}$

2.2 Approximate the number of modes of a fiber using a wavelength of 850 nm, a core radius of 80 µm and core index of 1.56 and cladding index of 1.50.

Answer: $M = \frac{4}{\pi^2}(2\pi\frac{80\ um}{850\ nm}\sqrt{n_1^2 - n_2^2})^2 = 26022$

## 2.1 Modal Delay

Arriving at equation 1 requires setting the *z* dependence of the propagating field proportional to $e^{j\beta z}$. The phase velocity for a harmonic field evolving in time as $e^{j\omega t}$ is then equal to

$$v_p = \frac{\omega}{\beta}. \quad (8)$$

However, the speed at which a pulse envelope centered at $\omega$ travels, also known as the group velocity, is given by

$$v_g = \frac{d\omega}{d\beta}. \tag{9}$$

This arises from the fact that the phase constant β is a function of frequency. From Fourier theory, even if a monochromatic source is used, the modulated pulse will contain frequencies other than the carrier frequency and each frequency will travel at slightly different phase velocities. This is the basis for understanding dispersion, which broadens time pulses after propagating over fiber.

The group velocity can be used to calculate the overall time delay of an optical pulse through a fiber by rearranging the equation into

$$t_g = \frac{L}{c}\left(N_{eff} - \lambda\frac{dN_{eff}}{d\lambda}\right), \tag{10}$$

where $N_{eff}$ is the mode effective index and calculated from $N_{eff} = \beta\lambda/2\pi$. Using the mode solver built in to OptiSystem the effective index can be found and then approximating the derivative with a difference the group delay can be calculated.

The goal of this next step is to demonstrate calculating the group delay of the stock step index multimode component at the carrier wavelength of 850 nm and length of 1 km. Start by placing the Measured-Index Multimode Fiber into a blank layout. Set the mode solver to use 850 nm as the solver wavelength.

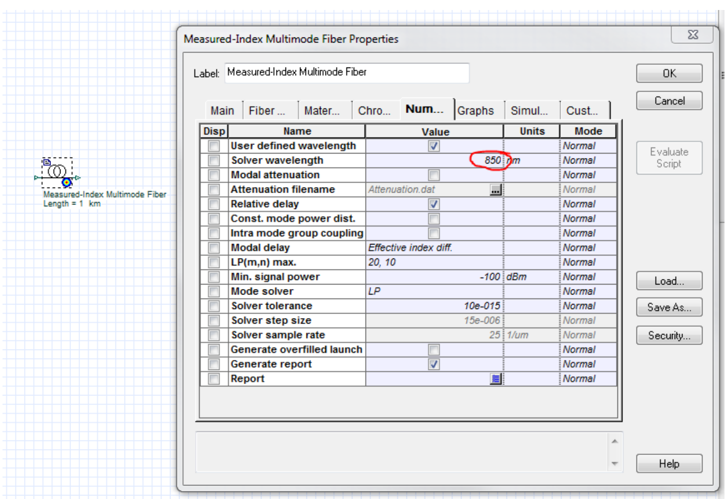



*Figure 2: Measured-Index Multimode Fiber component properties*

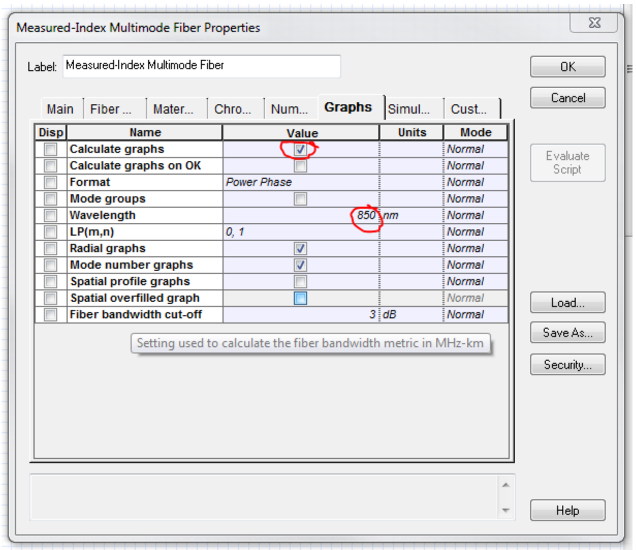



*Figure 3: Changing the graph options of the multimode fiber.*

In the Graphs tab, enable Calculate graphs and set the wavelength to 850 nm. Exit the component properties and run the simulation. From the Project Browser, you will be able to see the various graphs the component calculates, in particular double click the effective index graph and open it up. This graph shows the effective indices for different propagating modes starting on the left for the fundamental mode.

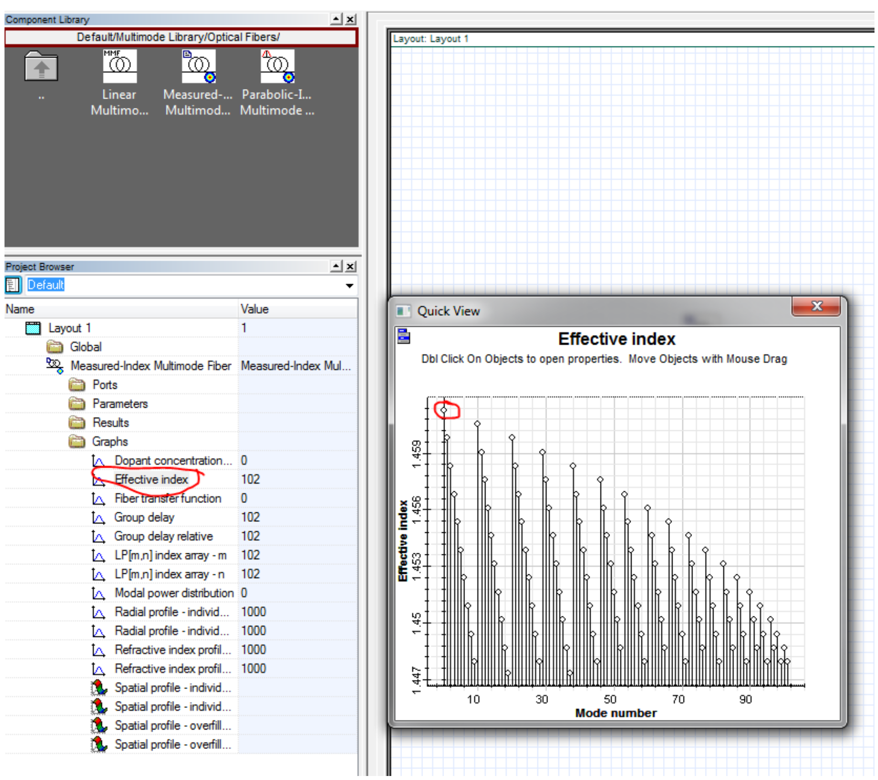



*Figure 4: Plotting the effective index as a function of the mode number.*

Double clicking the blue tab at the top left corner of the graph and choosing export data allows the creation of a data file with the exact numerical values. These files can then be imported into a different program to perform operations on them or to plot data, like Excel or Matlab.

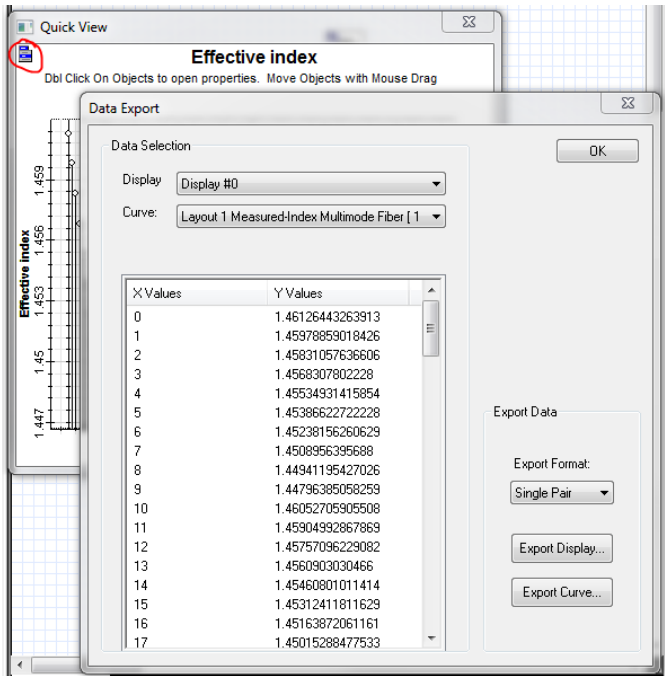



*Figure 5: Exporting data to a text file.*

Perform these steps two more times for wavelengths of 850.1 nm and 849.9 nm and match the table below relating wavelength to effective index of the $0^{th}$ mode.

| Wavelength | Effective Index |
|---|---|
| 849.9 nm | 1.46123755153895 |
| 850 nm | 1.46123746163327 |
| 850.1 nm | 1.46123737172757 |

Using these values and the central difference approximation for a derivative:

$$\frac{dF(x)}{dx} \approx \frac{F(x+\Delta x)-F(x-\Delta x)}{2\Delta x} = \frac{1.46123737172757-1.46123755153895}{0.2} = -8.990569E-7.$$

Now that the derivative is known all the remains is using the central wavelength and index of refraction in the group delay expression for the result.

$$t_g = \frac{1\ km}{299792\,\frac{km}{s}}(1.46123746163327 + 850\ nm\ \times 8.990569E-7\ nm^{-1}) = 4.87672\ \mu s$$

<u>Questions:</u>

2.1.1 Showing your work, find the group delay of the $10^{th}$ mode (Mode 9) with the default Measured-Index Multimode Fiber at the same wavelength.

Answer:

| Wavelength | Effective Index |
|---|---|
| 849.9 nm | 1.44751581124069 |
| 850 nm | 1.44751446007531 |
| 850.1 nm | 1.44751311049375 |

$$\frac{F(x+\Delta x)-F(x-\Delta x)}{2\Delta x} = -1.350373E-5$$

$$t_g = 4.86668\ \mu s$$

2.1.2 Two pulses, one travelling with the group velocity of mode 0 and the other with mode 9, propagate a distance of 100 km. Which pulse will reach the detector first and what will be the delay of the second pulse?

Answer: The mode 9 pulse will reach the detector first as it has a smaller group delay.

$$t_{g0} = 4.87672\frac{\mu s}{km} \times 100\ km = 487.672\ \mu s$$

$$t_{g9} = 4.86668\frac{\mu s}{km} \times 100\ km = 486.668\ \mu s$$

$$t_{gdelay} = 487.672\ \mu s\ - 486.668\ \mu s = 1.004\ \mu s$$

2.1.3 Generally when injecting light into a multimode fiber more than one mode is excited at the same time. If a pulse of light excites both the first and tenth mode, what will be the general tendency of the pulse as it propagates through the fiber?

Answer: The power in the different modes will travel at different group velocities resulting in a pulse envelope that broadens as it propagates through a multimode fiber.

## 2.2 Exciting Modes

Different spatial modes of a multimode fiber travel with different group velocities. If an input optical pulse excites more than a single mode the different group velocities will tend to broaden the pulse over the propagation distance. This in turn limits the length and bandwidth of multimode fiber based systems, since as the pulses broaden they may begin to overlap with adjacent bits.

The excited modes in a multimode fiber depend on the injected field. The amount of power coupled to specific mode can be calculated via an overlap integral. In special cases, like where the injected field is exactly the same as a specific fiber mode then all of the incident power will be coupled into that specific mode. In physical fibers, although one mode might be excited at the input of the fiber, power will spread to other modes through imperfections and bends.

Using the Spatial Visualizer in OptiSystem and attaching an input field at a wavelength of 850 nm to the fiber in the previous section, the fields supported by the fiber can be viewed. Before running the simulation, modify the Numerical tab in the Measured-Index Multimode Fiber by checking “Const. mode power dist.”. This will ignore the overlap integral and excite all modes with equal power, so that they can be viewed easily.

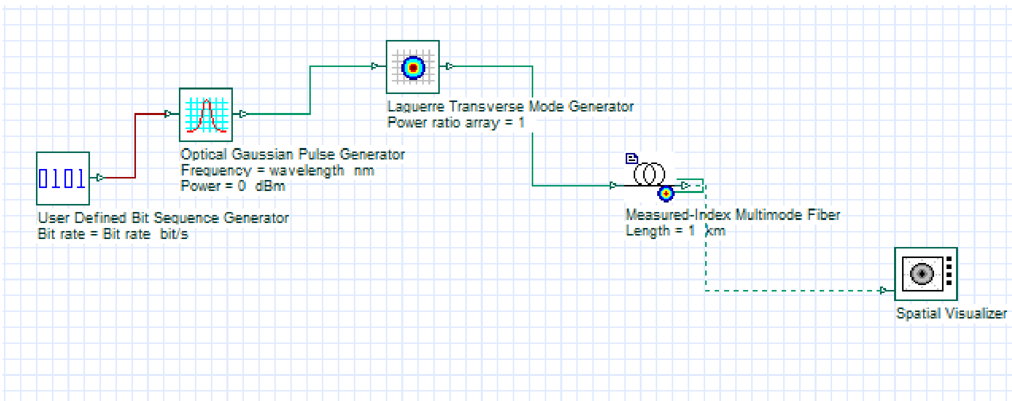



*Figure 6: Exciting modes in a multimode fiber layout.*

The first couple of modes of the fiber are given in Figure 7:

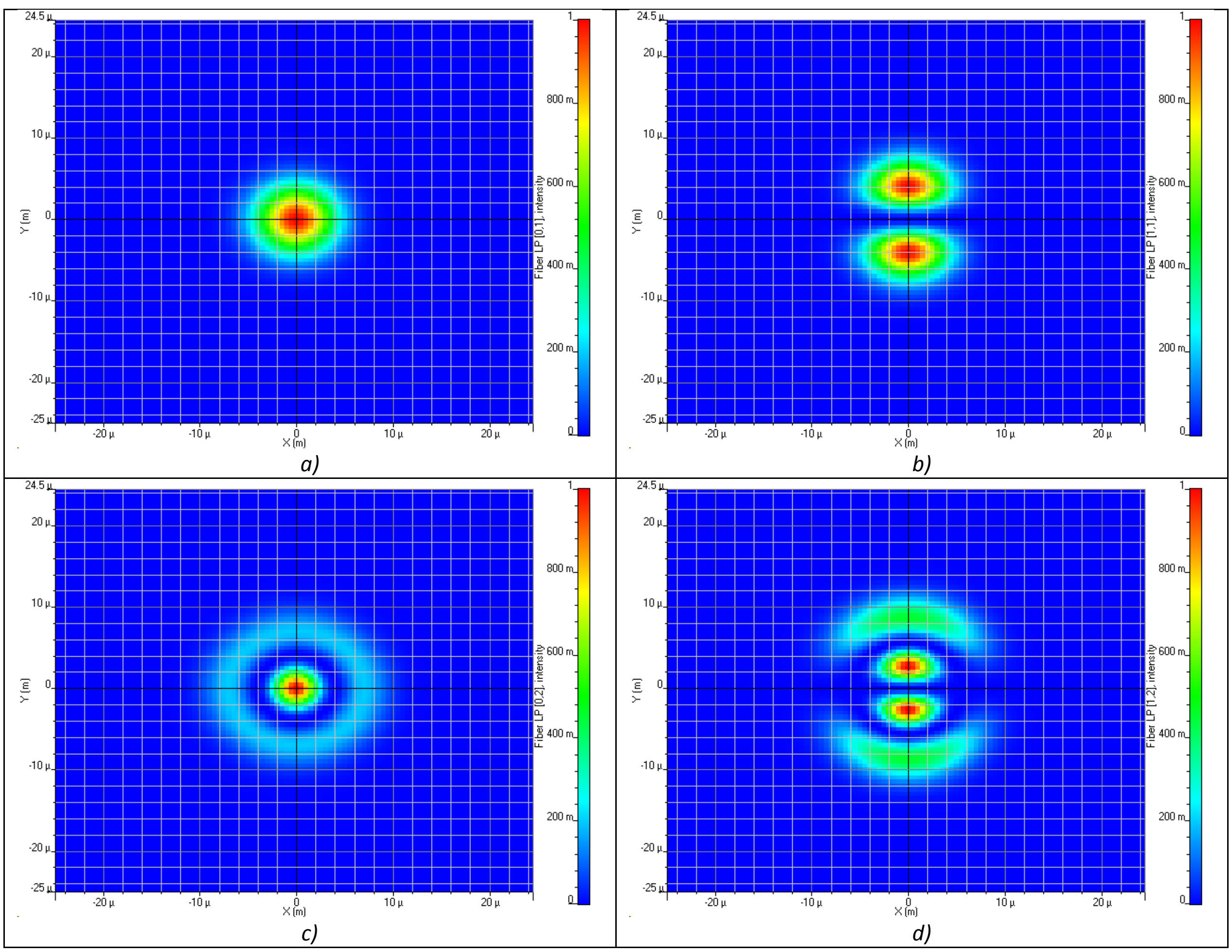



*Figure 7: Different $LP_{mn}$ modes of an optical fiber from top left to bottom right: LP01, LP02, LP11, LP12.*

These figures show the intensity of the $E_r$ field as a function of the transverse dimensions. The multimode component also calculates the coupling of power from the incident field into the different fiber modes. Navigate to the Numerical tab and at the bottom beside the Report parameter click on the small string editor button.

The coupling coefficient can be viewed for each $LP_{mn}$ mode. In this example, the coefficient is the same for each mode, since the option "Const. mode power dist." was checked. This information is also contained in one of the calculated graphs.

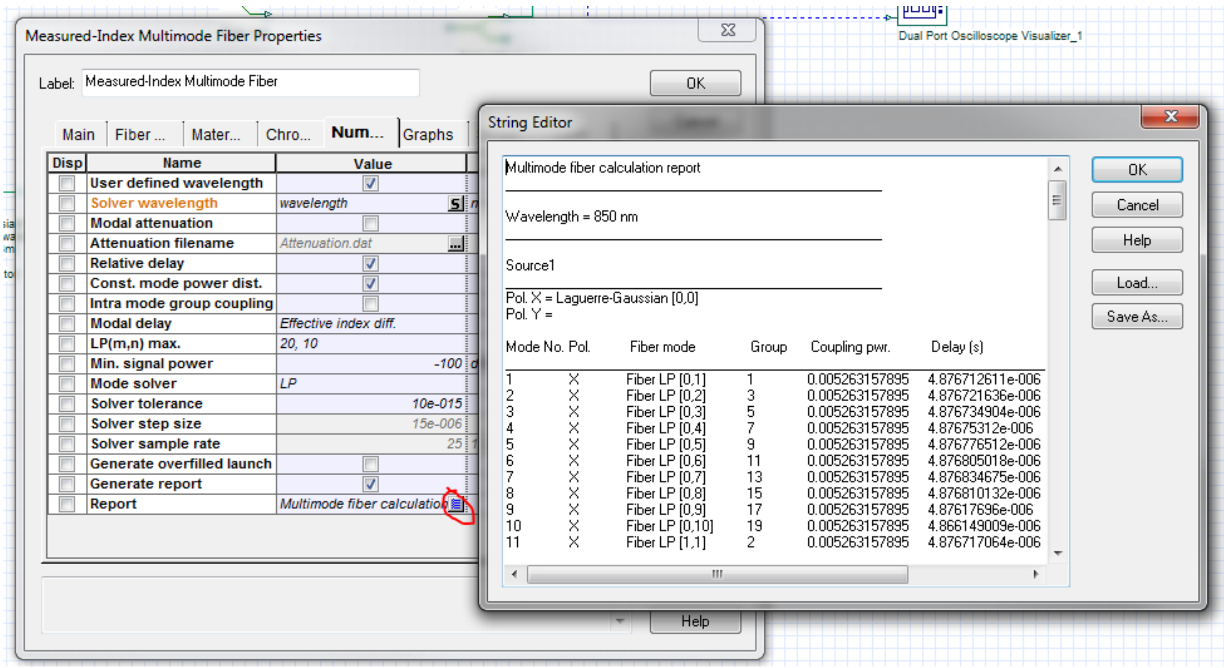


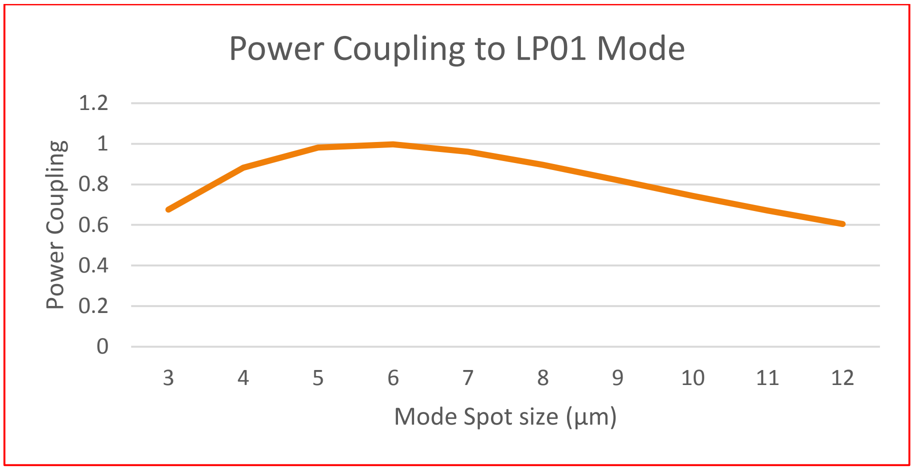

*Figure 8: Opening the multimode fiber calculation report.*

<u>Questions:</u>

2.2.1 Create a parameter sweep for the Laguerre Transverse Mode Generator to plot the coupling power coefficient for the $LP_{01}$ mode as a function of the injected mode spot size. Plot the relation and determine which injected spot size would minimize the modal dispersion.

Answer: The most effective way to limit modal dispersion is to couple as much power into a single mode as possible. Therefore using the spot size of 6 µm should minimize the modal dispersion.

| Mode Spot Size (µm) | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|
| Mode Coupling | 0.676 | 0.8817 | 0.9819 | 0.9978 | 0.9606 | 0.8959 | 0.8203 | 0.7436 | 0.6708 | 0.6041 |




# 3 Minimizing Modal Dispersion

Modal dispersion is unwanted in optical communication systems as it places a limit on the propagation distance and bandwidth. There are two popular ways that modal dispersion can be limited. Using a parabolic index fiber, where the index of refraction varies smoothly as a function of the radius, can be design to support modes all with very similar group velocities. The other direction is to eliminate multiple modes altogether, by using a single mode fiber. This can be achieved simply by reducing the core radius.

The default index of refraction profile is an Alpha or exponential distribution. Thus, to approximate the step index fiber set the Alpha parameter to 10e+009 and a suitable number of radial steps to resolve the small radius changes (>1000).

<u>Questions:</u>

3.1 Find the core radius that will modify the default Measured-Index Multimode Fiber to act as a single mode fiber with a carrier wavelength of 850 nm. Hint: Use the Graphs to view the different effective indices of the modes.

Answer: Should be approximately 1.55 µm radius

3.2 Using the dimensions of the fiber that give single mode operation, what is the associated V parameter?

Answer: $V = 2\pi \frac{1.55}{0.850} \sqrt{1.462^2 - 1.447^2} = 2.393$

The V parameter should be close to ~2.405, which is the analytic solution to of the requirement for single mode operation.

# 4 REPORT

In your lab report include the following:

- Brief overview of the background and theory.
- Answers to all pre lab questions, clearly showing your work.
- Brief description of the simulation method and setup, including screenshots.
- Final results including figures and discussion.

# 5 REFERENCES

[1] Saleh, Bahaa E. A., and Malvin Carl. Teich. *Fundamentals of Photonics*. New York: Wiley, 1991. Print.

[2] Agrawal, G. P. *Fiber-optic Communication Systems*. New York: Wiley, 1997. Print.