Gain Equalization for Few-Mode Fiber Amplifiers Beyond Two Propagating Mode Groups

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***Abstract*— We investigate gain equalization in few-mode fibers supporting more than two-mode groups at the signal wavelengths and find that a combination of pump control and doping profile optimization is required to equalize mode-dependent gain.**

***Index Terms*— Fiber optical communications, nonlinear optical signal processing.**

I. Introduction

SPACE-DIVISION multiplexing using few-mode fibers (FMF) has gained increasing interest as a means of increasing system capacity beyond single-mode fibers (SMF). Long-haul transmission using FMF was demonstrated in [1]. Ultimately, system cost is reduced by integrating components, and the feasibility of transmission systems based on FMF depends on the ability to fabricate devices like inline amplifiers and optical switches in FMF [2], [3]. In few-mode erbium-doped fiber amplifiers (FM-EDFA), gain equalization is of paramount importance. It has been proposed that controlling the mode content of the pump enables control of mode-dependent gain (MDG) [3]. Practical implementations of FM-EDFAs supporting the two lowest mode groups (LP_{01} and LP_{11}) was reported in [4], [5]. In this letter, we study the feasibility of FM-EDFAs supporting more than two signal mode groups [6].

II. Theory

Fig. 1 shows the assumed configuration for our FM-EDFA [3]. We assume that multi-moded signal and pump are generated using mode multiplexers. A dichoric mirror is used to combine their fields in free-space being launching into an erbium-doped few-mode fiber. Modal gain is controlled by varying the mode content of the pump. We assume the propagation constants of the signal modes and the pump modes to be sufficiently different such that an "intensity" model can be used to characterize the EDFA. The intensity model has been shown to accurately match experimental results in [7]. It can be shown that for a forward-pumped FM-EDFA, the power in signal mode $i(P_{s,i})$, the amplified spontaneous emission (ASE) power in signal mode $i(P_{ASE,i})$, and pump power in mode $j(P_{p,j})$ evolves as [3], [8]

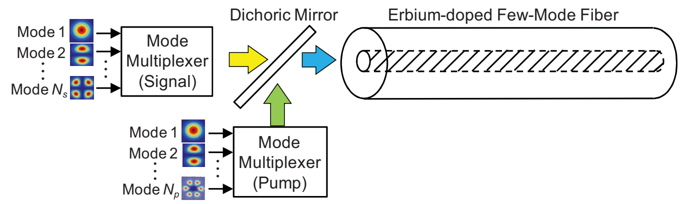

Fig. 1. Few-mode erbium-doped fiber amplifier with gain equalization using a multimoded pump.

$$\frac{dP_{s,i}}{dz} = P_{s,i}\int_0^{2\pi}\int_0^{a}\Gamma_{s,i}(r,\varphi)\Big[N_2(r,\varphi,z)\,\sigma_{es} - N_1(r,\varphi,z)\,\sigma_{as}\Big]\,r\,dr\,d\varphi \quad (1)$$

$$\frac{dP_{ASE,i}}{dz} = P_{ASE,i}\int_0^{2\pi}\int_0^{a}\Big\{\Gamma_{s,i}(r,\varphi)\Big[N_2(r,\varphi,z)\,\sigma_{es} - N_1(r,\varphi,z)\,\sigma_{as}\Big] + 2\sigma_{es}h\nu_s\Delta\nu N_2(r,\varphi,z)\Gamma_{s,i}(r,\varphi)\Big\}\,r\,dr\,d\varphi \quad (2)$$

$$\frac{dP_{p,j}}{dz} = -P_{p,j}\int_0^{2\pi}\int_0^{a}\Gamma_{p,j}(r,\varphi)\,N_1(r,\varphi,z)\,\sigma_{ap}\,r\,dr\,d\varphi, \quad (3)$$

where $\Gamma_{s,i}(r,\varphi)$ and $\Gamma_{p,j}(r,\varphi)$ are the normalized intensity patterns of the signal and pump modes, σ_{as} and σ_{ap} are the absorption cross-section areas at the signal and pump wavelengths, and σ_{es} and σ_{ep} are the emission cross-section areas. The steady-state upper- and lower-level population densities of the dopant $N_1(r,\varphi,z)$ and $N_2(r,\varphi,z)$ are given by

$$N_1(r,\varphi,z) = \frac{\left[\frac{1}{\tau} + \frac{1}{h\nu_s}\sum_{i=1}^{m_s}\left[P_{ASE,i}(z) + P_{s,i}(z)\right]\sigma_{es}\Gamma_{s,i}(r,\varphi)\right]N_0(r,\varphi)}{\left[\frac{1}{h\nu_s}\sum_{i=1}^{m_s}\left[P_{ASE,i}(z) + P_{s,i}(z)\right](\sigma_{es}+\sigma_{as})\Gamma_{s,i}(r,\varphi) + \frac{1}{\tau} + \frac{1}{h\nu_p}\sum_{j=1}^{m_p}P_{p,j}(z)\,\sigma_{ap}\Gamma_{p,j}(r,\varphi)\right]} \quad (4)$$

$$N_2(r,\varphi,z) = N_0(r,\varphi,z) - N_1(r,\varphi,z) \quad (5)$$

Manuscript received August 1, 2012; revised September 11, 2012; accepted September 12, 2012. Date of publication September 28, 2012; date of current version October 11, 2012.

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Digital Object Identifier 10.1109/LPT.2012.2219521

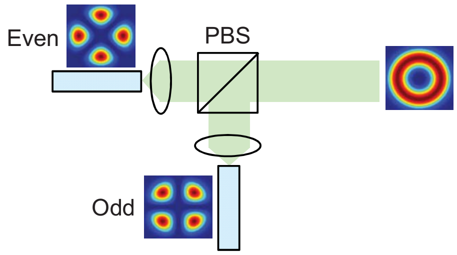


Fig. 2. Generating pump modes with intensity patterns having no azimuthal dependence.

with $N_0(r, \varphi, z)$ being the dopant density in the fiber. In general, the gain of a signal mode $G_i = P_{s,i}(L)/P_{s,i}(0)$ is dependent on the degree of overlap between the signal intensity, pump intensity, and doping profiles of the fiber. For example, it has been shown that in step-index fiber with a uniformly doped core carrying two mode groups, the signal LP_{01} mode has higher gain when pumped with LP_{01} and LP_{11}, while signal LP_{11} has higher gain when pumped with LP_{21} [3].

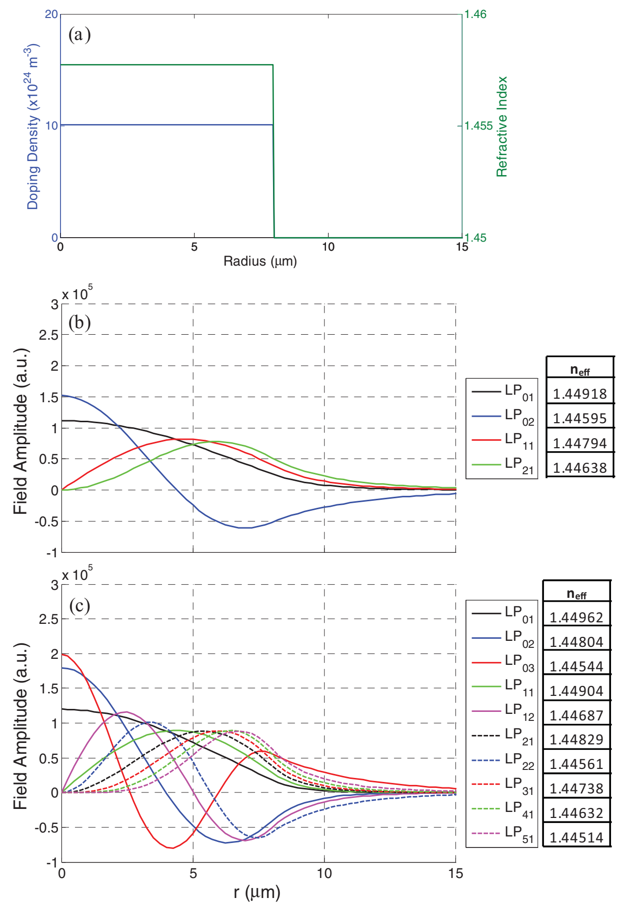
III. Simulation Results

To reduce the number of modal gains to be equalized, we do not consider polarization-dependent gain or mode-dependent gain between spatially degenerate modes. Instead, we consider all pump and signal modes to have ring-shaped intensity profiles that are independent of azimuthal angle (right figure in Fig 2). This assumption is valid for properly prepared pumps and signals. Firstly, population inversion does not respond fast enough to the symbol-rate fluctuations in signal intensity. If the degenerate spatial modes of the signal and their respective polarizations are modulated with independent data of equal power, the erbium atoms will respond to a time-averaged intensity pattern that is ring-shaped. For the pump, the scheme shown in Fig. 2 can be used where the two spatially degenerate modes are launched in orthogonal polarizations, and a polarization beam splitter (PBS) is used to combine their beams. The intensity pattern of the their output field will be "ring-shaped" with no azimuthal dependence.

We first consider a step-index fiber with a numerical aperture (NA) of 0.15 and a core radius of 8 μm [Fig. 3(a)]. This fiber carries four mode groups at a signal wavelength of $\lambda_s = 1550$ nm, and ten mode groups at a pump wavelength of $\lambda_p = 980$ nm. Fig. 3(b) and (c) show the radial components of the intensity profiles of the various signal and pump modes. The effective refractive indices of the modes are shown next to the legend. The maximum beat length $L_{beat,\max} = \lambda / \min(\Delta n_{eff})$ between any pair of signal modes is 3.6 mm ($LP_{02,s}$ and $LP_{21,s}$); while for the pump modes, the maximum beat length is 5.8 mm between $LP_{03,p}$ and $LP_{22,p}$. We assume a uniform doping density of 10^{25} m^{-3} inside the core [Fig. 3(a)]. The table in Fig. 3(d) shows the degree of overlap between signal and pump modes, which we defined as

$$\eta_{ij} = \int\int \Gamma_{s,i}(r, \varphi)\, \Gamma_{p,j}(r, \varphi)\; r\, dr\, d\varphi. \tag{6}$$

With the uniform doping assumption, a larger overlap integral indicates higher gain. Our goal is to find for each



(d)		Pump Mode									
		01	02	03	11	12	21	22	31	41	51
Signal Mode	01	**7.48**	6.25	4.95	5.34	5.67	4.12	4.64	3.30	2.70	2.17
	02	5.33	**7.71**	5.44	2.55	4.92	2.23	2.79	2.38	2.57	2.66
	11	4.63	3.32	2.79	5.26	3.65	**5.08**	3.76	4.69	4.22	3.67
	21	2.98	2.95	2.31	4.25	2.78	4.72	2.84	4.81	4.68	**4.34**

Fig. 3. (a) Refractive index and doping profile for a step-index few-mode EDFA supporting four-mode groups at the signal wavelength. Radial modal profiles at (b) $\lambda_s = 1550$ nm and (c) $\lambda_p = 980$ nm. (d) Degree of overlap between pump and signal modes.

signal mode i a pump mode j that most favorably amplifies i relative to other modes k. We thus select pump mode j with the property that $j = \max_{l}\{\eta_{il} - \max_{k \neq i}\{\eta_{kl}\}\}$, as this maximizes the gain differential between the desired signal mode i and the next most favorably amplified signal mode k. The ideal pumping strategy is shown by the bolded entries in Fig. 3(d), with pump modes $LP_{01,p}$, $LP_{02,p}$, $LP_{21,p}$ and $LP_{51,p}$ giving the best differential gains for signal modes $LP_{01,s}$, $LP_{02,s}$, $LP_{11,s}$ and $LP_{21,s}$, respectively.

With the set of pump modes chosen, Fig. 4(a) shows the pump powers required to equalize the gains of the signal modes to within 0.1 dB of the horizontal abscissa. We have assumed the use of 10 m of the FM-EDFA shown in Fig. 3(a), which is much longer than the maximum beat length between signal and pump modes, enabling the use of the intensity model. We have further assumed that the input mode-division multiplexed (MDM) signal has -13 dBm power in each mode-polarization at the signal wavelength: i.e., a total power of

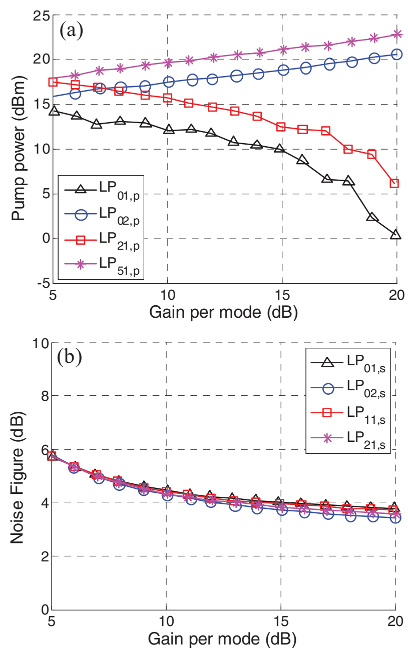


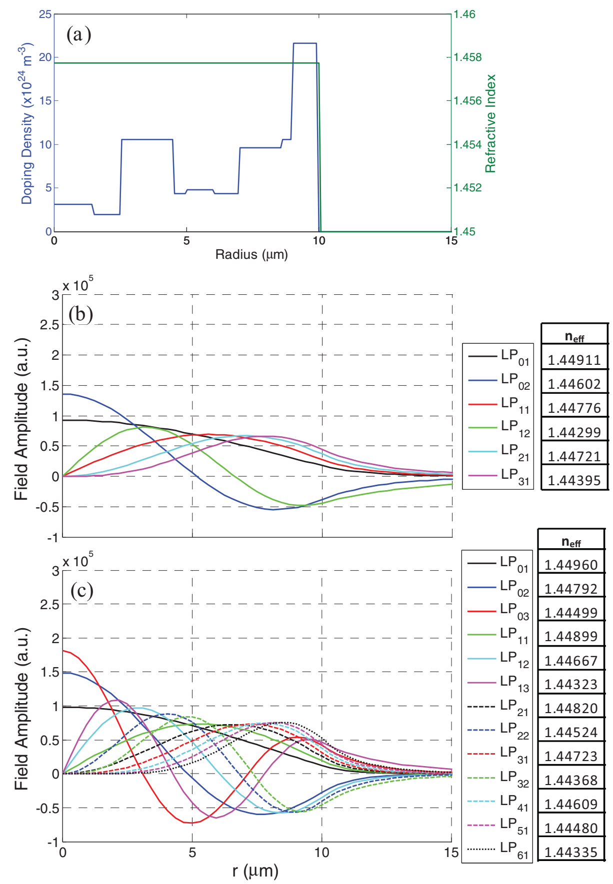
Fig. 4. (a) Pump power required to obtain equalized gain in each mode group of a four-mode group FM-EDFA. (b) Noise figure of the FM-EDFA.

−10 dBm in each spatially non-degenerate mode (LP_{01}, LP_{02}) and −7 dBm in each spatially degenerate mode (LP_{11}, LP_{21}).

The large spread in pump powers observed at high gain is due to the gain metric being ill-conditioned; while pump $LP_{21,p}$ barely achieves higher gain for $LP_{11,s}$ compared with the other signal modes, pump $LP_{01,p}$ produces much higher gain for its favored $LP_{01,s}$ mode, but little gain for the two highest-order signal modes. Fig. 4(a) reveals that at high gain, most of the pump power is in $LP_{02,p}$ and $LP_{51,p}$ which preferentially amplifies the two lowest and two highest signal modes. Small amounts of $LP_{01,p}$ and $LP_{21,p}$ are only needed to fine tune MDG. At gains above 20 dB, saturation effects cause the $LP_{02,p}$ pump to be such that even with no $LP_{01,p}$ pump, the amplifier cannot produce higher gain for $LP_{02,s}$ compared with $LP_{01,s}$. Hence, MDG cannot be equalized below the 0.1 dB tolerance at higher target gains. Fig. 4(b) shows the noise figure for the signal modes associated with the pumping scheme shown in Fig. 4(a). It is observed that the noise figures decrease asymptotically to the 3 dB limit with increasing gain.

We further explore gain equalization in a fiber carrying six mode groups, whose refractive and doping index profiles are shown in Fig. 5. We assume that the erbium dopant has minimal impact on refractive index compared with the dopant (e.g., aluminum) used to control refractive index. Hence, it is assumed that the dopant concentration profile can be controlled independently of the refractive index profile. For fair comparison with the previous four-mode fiber, we assume a step-index refractive index profile with NA = 0.15 as before. The core radius was set to 10 μm. Fig. 5(b) and (c) shows the radial profiles of the six signal modes and thirteen pump modes at 1550 nm and 980 nm, along with n_{eff} for the various modes. The maximum beat length between pairs of signal modes is 2.8 mm between $LP_{11,s}$ and $LP_{21,s}$; and for the pump modes, the maximum beat length is 8.2 mm between $LP_{13,p}$ and $LP_{61,p}$. For fair power efficiency comparison with the previous fiber, we set the average dopant concentration in the core to be 10^{25} m^{-3} and the fiber length to be 10 m, the same as the previous fiber. We also choose the pump modes by the same criterion as before, and the strategy is shown by the bolded entries in Fig. 5(d). In contrast with the four-mode fiber, we found that modal gain cannot be equalized using a simple step-index doping profile. Instead, the dopant profile had to be adjusted heuristically to give more gain to the signal modes that have relatively poor overlap with the pump modes [Fig. 5(d)]. Fig. 5(a) shows a doping profile that enabled the equalization of MDG over a wide range of target gain values.

Fig. 6(a) shows the pump powers required to equalize the gains of the signal modes to within 0.12 dB of the horizontal



(d)		Pump Mode												
		01	02	03	11	12	13	21	22	31	32	41	51	61
Signal Mode	01	**5.13**	4.34	4.09	3.64	4.03	3.64	2.81	3.56	2.27	2.01	1.88	1.60	1.29
	02	4.12	**6.12**	4.97	2.03	3.83	3.95	1.83	2.26	1.96	2.43	2.10	2.19	2.20
	11	3.28	2.32	2.31	**3.69**	2.63	2.24	3.55	2.91	3.28	2.52	2.99	2.71	2.34
	12	3.16	2.50	2.30	2.15	3.51	2.52	1.55	**3.21**	1.34	2.98	1.33	1.39	1.51
	21	2.26	2.15	1.93	3.17	2.06	1.81	3.50	2.25	**3.55**	1.47	3.47	3.31	3.02
	31	1.59	2.09	1.71	2.56	1.88	1.61	3.09	1.90	3.36	3.06	3.47	3.47	**3.33**

Fig. 5. (a) Refractive index and doping profile for a step-index few-mode EDFA supporting six-mode groups at the signal wavelength. Radial modal profiles at (b) λ_s = 1550 nm. (c) λ_p = 980 nm. (d) Degree of overlap between pump and signal modes.

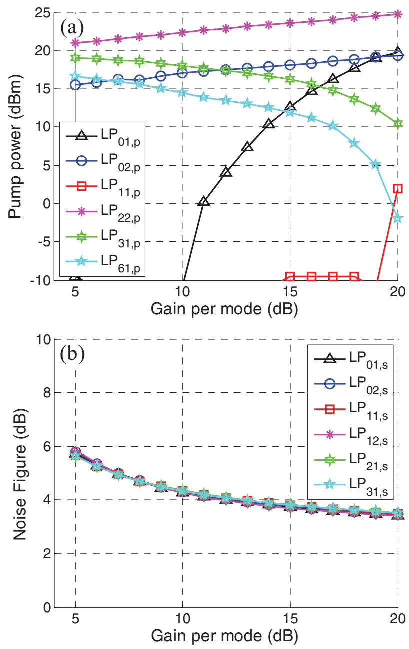


Fig. 6. (a) Pump power required to obtain equalized gain in each mode group of a six-mode group FM-EDFA. (b) Noise figure of the FM-EDFA.

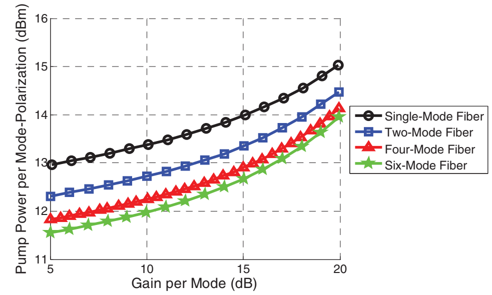


Fig. 7. Total pump power required to obtain equalized gain in each mode.

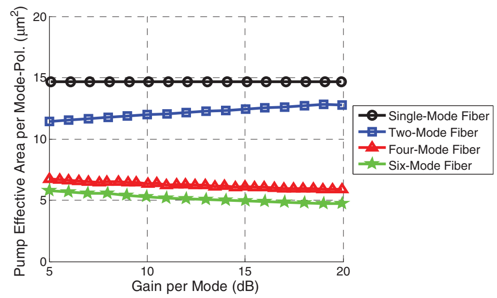


Fig. 8. Pump effective areas for various EDFAs supporting different number of modes.

abscissa. It is again observed that some pumps have little power at the highest and lowest simulated gains. In fact, for target gain values below 8 dB, we were unable to equalize the individual modal gains to within the 0.12 dB tolerance, and these results are shown as shaded regions in Fig. 6(a). The noise figures of the various modes are shown in Fig. 6(b), indicating that once again, the noise figures of all modes approach the 3 dB theoretical limit at high gain.

Since in a few-mode fiber, all pump and signal modes are co-located, the amount of pump power in the undoped cladding region is reduced, leading to better pump efficiency. It has been shown that few-mode amplifiers can achieve efficiency improvements of around 30% per bit compared with a parallel $N \times$ SMF system [2]. Using the pump powers found in Figs. 4 and 6, we computed the pump requirement per non-degenerate mode-polarization necessary to obtain a target gain. The results are shown in Fig. 7. For comparison, results are also shown for single-mode (2 mode-polarizations) and two-mode-group (6 mode-polarizations) cases assuming step-index fibers with uniform doping within the core, with NA = 0.15 and core radii of 3.8 and 6 μm, respectively. As expected, we observe that a fiber supporting a larger number of mode groups require less pump power per signal mode-polarization. Fig. 8 shows the "pump effective area" for the various EDFAs, which we define as

$$\bar{A}_{eff,p} = \sum_{j=1}^{N_p} P_{p,j} \Bigg/ \sum_{j=1}^{N_p} P_{p,j} A_{eff,p,j}^{-1}, \tag{7}$$

where $P_{p,j}$ and $A_{eff,p,j}$ are the pump power and effective areas of pump mode j, and N_p is the number of pump modes. $\bar{A}_{eff,p}$ is an effective cross-sectional area over which the pump energy is concentrated. Compared with the parallel $N \times$ SMF case, it is observed that as the number of mode groups increases, the pump becomes concentrated in a smaller effective area. The higher pump power density results in improved efficiency.

IV. Conclusion

We have studied gain equalization in FM-EDFAs with more than two mode groups. We found that MDG control becomes increasingly difficult, and that optimization of the doping profile is required in conjunction with pump control to equalize MDG for fibers having large (>4) number of mode groups.

References

[1] S. Randel, *et al.*, "Mode-multiplexed 6×20-GBd QPSK transmission over 1200-km DGD-compensated few-mode fiber," in *Proc. OFC/NFOEC 2012*, Los Angeles, CA, pp. 1–3, paper PDP5C.5.

[2] P. M. Krummrich, "Optical amplifiers for multimode/multicore transmission," in *Proc. OFC/NFOEC 2012*, Los Angeles, CA, pp. 1–3, paper OW1D.1.

[3] N. Bai, *et al.*, "Multimode fiber amplifiers with tunable modal gain using a reconfigurable multimode pump," *Opt. Express*, vol. 19, no. 17, pp. 16601–16611, 2011.

[4] N. Bai, *et al.*, "Mode-division multiplexed transmission with inline few-mode fiber amplifier," *Opt. Express*, vol. 20, no. 3, pp. 2668–2680, 2012.

[5] Y. Yung, "First demonstration of multimode amplifier for spatial division multiplexed systems," in *Proc. ECOC 2011*, Geneva, Switzerland, pp. 1–2, paper Th.13.K.4.

[6] E. Ip, "Gain equalization for few-mode fiber amplifiers with more than two propagating mode groups," in *Proc. IEEE Summer Topical*, Seattle, WA, Jul. 2012, pp. 224–225, paper WC2.4.

[7] N. Bai, *et al.*, "Experimental study on a multimode erbium-doped fiber amplifier with reconfigurable multimode pump," in *Proc. OFC 2012*, Los Angeles, CA, pp. 1–3, paper OW1D.3.

[8] M. Gong, *et al.*, "Numerical modeling of transverse mode competition in strongly pumped multimode fiber lasers and amplifiers," *Opt. Express*, vol. 15, no. 6, pp. 3236–3246, 2007.