Single Mode Optical Fiber - Dispersion

1 OBJECTIVE

Characterize analytically and through simulation the effects of dispersion on optical systems.

2 PRE-LAB

A single mode fiber, as the name implies, supports only a single transverse mode. The benefits of supporting only a single mode is that modal dispersion is eliminated since all pulses travel with the same modal group velocity. The fundamental mode of optical fibers is shown below in figure 1. For a core/cladding fiber structure the field follows a Bessel function, which can be approximated quite well by a Gaussian field for weakly guiding structures.
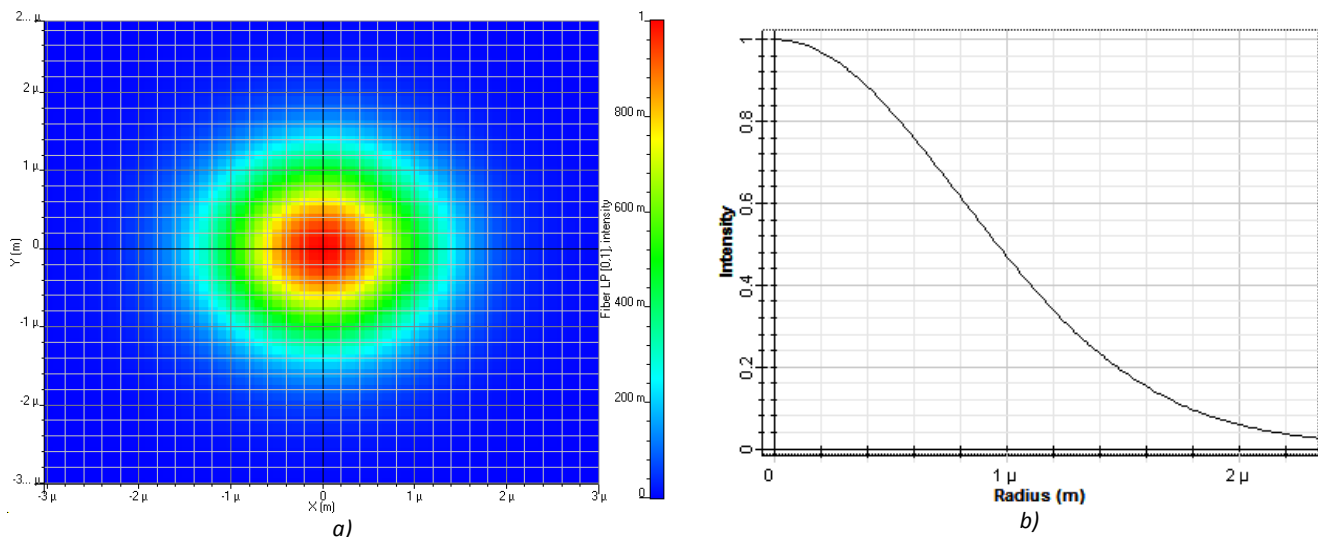

Figure 1: The fundamental mode of a single mode fiber: a) The intensity profile in the traverse direction. b) Intensity as a function of radius.

Questions:

      2.1   How many orthogonal modes exist in a single mode fiber? Hint: Consider the rotational symmetry of the fiber and vector field attributes of the electric field.

2.1 CHROMATIC DISPERSION

Although there is no modal dispersion between different propagating modes, dispersion has not been completely eliminated. For a single mode fiber, the dominant forms of dispersion are material and waveguide dispersion. Material dispersion stems from the frequency dependence of the index of refraction, whereas the waveguide dispersion arises from the frequency dependence of the propagation

constant for the fundamental mode. Together these two related effects force a frequency dependence for the group velocity of a pulse.

If the derivative of the group velocity with respect to the frequency, $v_g = \frac{d\omega}{d\beta}$, is non-zero then a time pulse will broaden through propagation as the different spectral components will arrive at different times. The pulse broadening, ΔT, is related to the derivative of the phase constant, β by:

$$\Delta T = L\frac{d^2\beta}{d\omega^2}\Delta\omega = L\beta_2\Delta\omega, \tag{1}$$

where $\Delta\omega$ is the spectral width of the pulse. In another form the pulse broadening can be given in terms of the wavelength range:

$$\Delta T = LD\Delta\lambda, \tag{2}$$

where D is called the dispersion parameter.

$$D = -\frac{2\pi c}{\lambda^2}\beta_2 \tag{3}$$

If a fiber has a positive dispersion parameter it is called anomalous dispersion and higher frequency components travel faster. If the dispersion parameter is negative it is called normal dispersion and lower frequency components travel faster.

Below is an example of a dispersion broadened pulse, notice that in addition to the pulse width increasing, so does the peak power decrease.
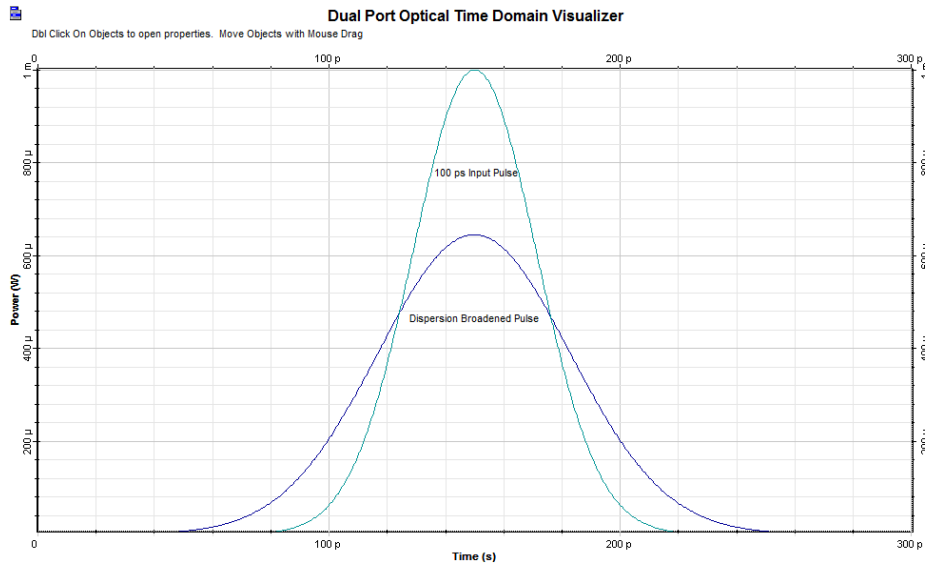

Figure 2: Dispersion Broadening of a 100 ps time pulse after 50 km propagation through an anomalous dispersive fiber.

Dispersion is quite simple to model by itself. In fact, its effect on a pulse can be modelled by the differential equation:

$$\frac{\partial A}{\partial z} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} = 0. \tag{4}$$

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This equation is set in the reference frame of the moving pulse with pulse shape $A(z, t)$. The higher order dispersion term β_3 is ignored for now, which makes solving the equation much simpler.

Questions:

    2.1.1   Calculate the time delay after 50 km of propagation between two frequency components separated by 2 nm and with a dispersion parameter of 16.75 ps/km ·nm

    2.1.2   Derive an expression for $\Delta\omega$ as a function of $\Delta\lambda$ using $c = \frac{\omega}{2\pi}\lambda$.

    2.1.3   For an optical pulse of constant phase propagating in a normal dispersion fiber. Which frequency components will be detected first to at the leading edge of the pulse?

2.2 CHIRPING OF PULSES

Optical signals can be represented by complex envelopes modulated at a carrier frequency, ω. That is to say, the optical pulses can be represented mathematically as:

$$E_x(t) = |A(t)|e^{j\varphi(t)}e^{j\omega t}, \tag{5}$$

where $A(t)$ is a complex valued function and:

$$\varphi(t) = arg[A(t)]. \tag{6}$$

A simple Gaussian pulse would be of the form:

$$E_x(t) = e^{-(t)^2}e^{j\omega t}. \tag{7}$$

Below is an example of what the magnitude and real part of these functions would look like. The $\varphi(t)$ value is constant 0 for this signal. The real part of the signal would be the actual electric field value, but representing the field as an envelope and removing the explicit representation of the carrier frequency is a useful tool, for example in the description of chromatic dispersion in the previous chapter.
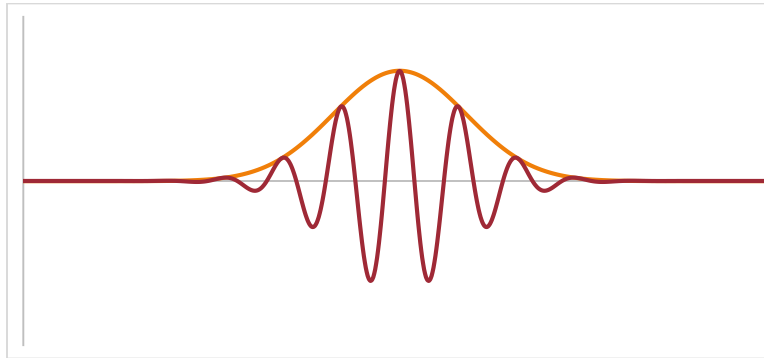

Figure 3: Gaussian pulse envelope with the real part of the signal shown as well.

Instead of a purely real Gaussian pulse, we will introduce a time dependent phase $\varphi(t)$. This will be represented by the equation:

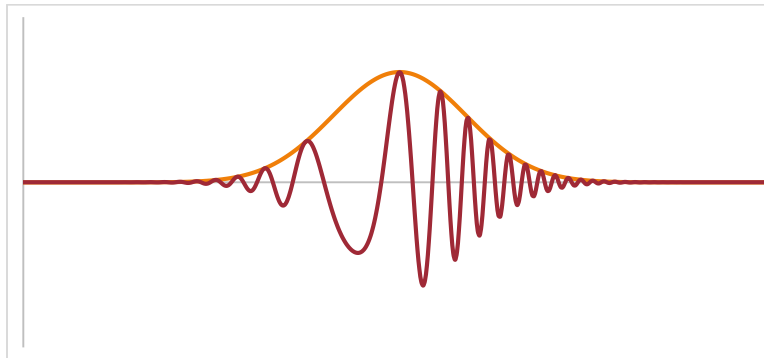$$E_x(t) = e^{-(t)^2} e^{jCt^2} e^{j\omega t}. \tag{8}$$

Figure 4: A chirped Gaussian envelope clearly showing the change in the carrier frequency as the phase changes over the pulse.

This is effect is known as chirp, described by the constant *C*, the chirp parameter. This time dependent phase results in a linear change in the instantaneous frequency. Changing the phase of the envelope has no effect on the magnitude of the original envelope, but it does change the frequency content.

Questions:

    2.1.1   From the expression for the simple chirped Gaussian pulse, find the instantaneous frequency $\omega(t)$ which can be derived from the derivative of the phase with respect to time.

3   EFFECT OF DISPERSION ON PULSES

Short pulses in optical fiber are broadened by group velocity dispersion. Naturally in OptiSystem it is possible to simulate these effects on pulses. Setting up a layout, as in the example below the effects can be investigated in more detail.

3.1   BROADENING OF GAUSSIAN PULSES

Only a few changes to the default parameters is needed to begin the simulation. Place a User Defined Bit Sequence Generator, setting the bit sequence to "0100000000000000" and the Sequence length in the Layout parameters to 16 bits. This allows for the injection of a single pulse. In the Optical Fiber disable the Attenuation effect, Third-order Dispersion and Self-phase modulation. In the PMD tab, set the Birefringence type to "Deterministic" and set Differential group delay to 0. This creates an optical fiber model that only includes group velocity dispersion. For an increase in simulation accuracy setting the Samples per bit to 256 will be large enough to provide very accurate simulations while keep the simulation time low.

- User Defined Bit Sequence Generator   Transmitters Library/Bit Sequence Generators
- Optical Gaussian Pulse Generator   Transmitters Library/Pulse Generators/Optical
- Fork 1x2   Tools Library
- Optical Power Detector   Receivers Library/Photodetectors

- Time Delay   Passives Library/Optical
- Clock Recovery   Receivers Library/Regenerators
- Optical Time Domain Visualizer   Visualizer Library/Optical
- Optical Spectrum Analyzer   Visualizer Library/Optical
- Dual Port Optical Time Domain Visualizer   Visualizer Library/Compare
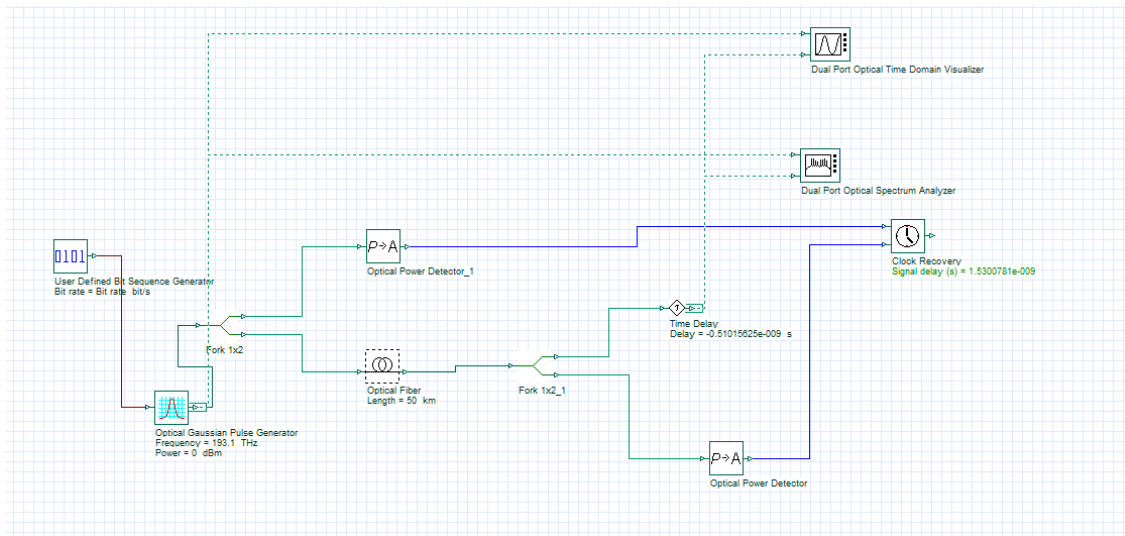- Dual Port Optical Spectrum Analyzer   Visualizer Library/Compare

Figure 5: Layout for simulating dispersion.

The group velocity dispersion will also cause a slight delay to the entire envelope in relation to the input pulse, so the Clock Recovery component is used in conjunction with the Time Delay component to re-center the pulse with the input. Using markers in the time domain graph it is straightforward to calculate the FWHM of the pulses.

Questions:

3.1.1   Using this setup and the default setting for group velocity dispersion, plot the T_{FWHM} as a function of distance. Describe the relation.

3.1.2   Using this setup and the default setting for group velocity dispersion, plot the ω_{FWHM} as a function of distance. Describe the relation.

3.1.3   Does dispersion modify the magnitudes of the frequency components? Does it affect the phase or time delay of the frequency components?

3.2   BROADENING OF HIGHER ORDER GAUSSIAN PULSES

3.3.1   Change the order of the Gaussian pulse to 4 and plot the T_{FWHM} as a function of distance. Describe the relation.

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3.3.2   Explain the strange behavior of the full width half maximum for short distances. Is the T_{FWHM} the best method for measuring spreading? Compared to the first order Gaussian the fourth order spreads much more quickly, by comparing the spectrums of both explain why.

3.3   BROADENING OF CHIRPED GAUSSIAN PULSES

The chirp parameter for a Gaussian pulse of pulse width $T_0 \approx {T_{FWHM}}/{1.665}$ is defined as the rate of change of the instantaneous frequency multiplied by $T_0{}^2$ with the equation for the instantaneous frequency being:

$$\delta\omega = \frac{C}{T_0{}^2}t. \qquad (9)$$

3.3.3   Plot T_{FWHM} for a regular and chirped Gaussian pulse as a function of distance. In the Optical Gaussian Pulse Generator set the Chirp factor in the Chirp tab to 1 rad/s. Describe the difference.

3.3.4   Plot the chirp for a regular and chirped Gaussian pulse as a function of distance. In the Optical Gaussian Pulse Generator set the Chirp factor in the Chirp tab to 1 rad/s. Describe the difference.

4   REPORT

In your lab report include the following:

- Brief overview of the background and theory.
- Answers to all pre lab questions, clearly showing your work.
- Brief description of the simulation method and setup, including screenshots.
- Final results including figures and discussion.

5   REFERENCES

[1]   Agrawal, G. P. *Fiber-optic Communication Systems*. New York: Wiley, 1997. Print.

[2]   Saleh, Bahaa E. A., and Malvin Carl. Teich. *Fundamentals of Photonics*. New York: Wiley, 1991. Print.